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$D = 6$, $N = 2$, $F(4)$-supergravity with supersymmetric *de Sitter* background

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Abstract: We show that there exists a supersymmetric *de Sitter* background for the $D = 6$, $N = 2$, $F(4)$ supergravity preserving the compact R-symmetry and gauging with respect to the conventional *anti de Sitter* version of the theory. We construct the gauged matter coupled $F(4)$ *de Sitter* supergravity explicitly and show that it contains ghosts in the vector sector.

Keywords: AdS-CFT and dS-CFT Correspondence, Supergravity Models, Differential and Algebraic Geometry.

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1. Introduction

Among the various supergravity theories with an arbitrary number $N \leq 32$ of supersymmetry charges and in arbitrary dimensions $D \leq 11$, the $D = 6$, $N = 2$, $F(4)$ supergravity plays a special role. First of all it is based on an exceptional supergroup not belonging to the $\mathrm{OSp}(n|m)$ or $\mathrm{SU}(n|m)$ series, namely the $F(4)$ supergroup, according to the classification given in references [1, 2]. Besides, $F(4)$ appears to be the only supergroup admitting two real sections whose bosonic (even) generators span either the algebra of $\mathrm{SO}(2,5) \otimes \mathrm{SU}(2)$ or $\mathrm{SO}(1,6) \otimes \mathrm{SU}(2)$, namely two product groups whose first factor describes the isometry group of *anti de Sitter* or *de Sitter* space-time respectively (see for example the classification given in references [3, 4]).

The above property points at the possibility of constructing a $D = 6$, $N = 2$ (sixteen supersymmetries) supergravity theory, admitting a supersymmetric vacuum whose space-time metric is of *de Sitter* type, besides the conventional possibility discussed in references [5, 6, 7] of having an *anti de Sitter* supersymmetric vacuum. Actually, the latter theory was first constructed by Romans [5] for the pure supergravity multiplet, using the Noether approach, and it was found that the theory admits a supersymmetric *anti de Sitter* (AdS_6) background when the gauge coupling constant g and the AdS_6 radius $R = (2m)^{-1}$ satisfy the relation $g = 3m$. Since Romans' theory is unique by construction, it is clear that an $F(4)$ supergravity admitting a *de Sitter* background, can only exist if it differs from Romans' theory; in this respect, we will find that a supersymmetric *de Sitter* background implies necessarily the presence of ghosts.

It is a well known fact that in conventional supergravities, quite generally, *de Sitter* configurations break supersymmetry completely [8]. Actually, several examples have been given recently of "variant" supergravity theories admitting a supersymmetric *de Sitter* backgrounds which, besides possessing non compact R-symmetry groups are also plagued with the presence of ghosts [9]. These "variant" theories, are string theories in $D = 10$ or

M-theory in $D = 11$, which can have the usual Minkowski signature or a non standard signature of space-time. They are usually obtained by applying T-duality transformations on timelike circles or lorentzian tori starting from conventional theories. Generally the theories thus obtained have ghosts, but in reference [10] it was argued that the corresponding complete string theory could possess some extra symmetry which could eliminate these spurious degrees of freedom. In our case we find that the *de Sitter* $F(4)$ supergravity does indeed show ghosts in the vector sector, however it shows the peculiarity that the R-symmetry group remains compact, namely it is $\mathrm{SU}(2)$ as in the sister *anti de Sitter* theory of the conventional type [5, 6, 7]. Furthermore, the gauge group of the gauged theory is exactly the same as for the sister AdS_6 theory, differently from what happens in the variant theories considered in the literature.

In references [6, 7], Romans' theory was reformulated in superspace and the important mechanisms of generation of a supersymmetric AdS_6 configuration and of the Higgs phenomenon for the generation of a mass term for the two-form sitting in the gravitation multiplet was explained in purely algebraic terms, by studying the superspace algebra in its dual form. Subsequently the theory was coupled to an arbitrary number n of vector multiplets and the gauging of the gauge group $\mathrm{SU}(2) \otimes \mathcal{G}$, with $\mathcal{G} \subset \mathrm{SO}(n)$, n-dimensional compact subgroup, was performed. The approach we follow for the construction of the $F(4)$ *de Sitter* supergravity is quite analogous to that used in references [6, 7].

We start from the Maurer-Cartan equations (MCE) equations of the supergroup $F^t(4)$ containing the proper subgroups $\mathrm{SO}(t, 7-t)$, with $t = 1, 2$. Thus $F^1(4)$ contains the *de Sitter* group and $F^2(4)$ contains the *anti de Sitter* group in six dimensions.

It is then straightforward to pull-back the MCE of $F^t(4)$ to the MCE of the coset $F^t(4)/\,\mathrm{SO}(1,5) \otimes \mathrm{SU}(2)$ which describes supersymmetric configurations of vacua whose metric is of *de Sitter* or *anti de Sitter* type, for $t = 1$, $t = 2$ respectively. Once this have been obtained, the construction of the relevant supergravity theory can be performed in the usual way, defining the supercurvatures out of the vacuum and by using the Bianchi identities in superspace, as it was extensively discuss in reference [6] in the $F^2(4)$ case.

Since $F^1(4)$ and $F^2(4)$ are two different real sections of the same complex supergroup $F(4)$, the discussion of the differences between them has some subtleties related to the discussion of the different reality properties of the bilinear fermion currents appearing in the two cases. Even if rather technical, the discussion of these points is quite essential in order to understand the relations between the two theories and the appearance of ghosts in the *de Sitter* case.

The plan of the paper is the following.

In section 2 we construct the MCE for the two cosets and their extension to a Free Differential Algebra (FDA), in order to include in the game the 2-form $B_{\mu\nu}$ and the extra $\mathrm{SU}(2)$ singlet vector A_μ sitting in the gravitational supermultiplet. A full discussion of the reality properties of the FDA is also given.

In section 3 we give the explicit form of the $F^1(4)$ supergravity theory admitting a *de Sitter* supersymmetric vacuum, its supersymmetry transformation laws and the relevant terms of the lagrangian, showing in an explicit way the appearance of ghosts in the vector sector.

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In appendix A, we describe the modifications of the $F^1(4)$ theory when the coupling to n vector multiplets and the gauging is performed, while appendix B describes an alternative approach to the construction of the $F^1(4)$ theory starting the $F^2(4)$ FDA and performing a suitable map.

2. Construction of $D = 6$ supersymmetric *de Sitter* background

To obtain a $D = 6$ supergravity with a supersymmetric *de Sitter* background, we begin to consider the complex superalgebra $F(4)$; it has two real sections, denoted by $F^t(4)$, $t = 1, 2$ containing as even part (bosonic subalgebra)$\mathrm{SO}(t, 7-t) \times \mathrm{SU}(2)$, that is the *de Sitter* group $\mathrm{SO}(1, 6)$ and the *anti de Sitter* group $\mathrm{SO}(2, 5)$ respectively, times the R-symmetry group $\mathrm{SU}(2)$.

The $F^t(4)$ superalgebra has the following form

$$\begin{aligned}
[M_{\hat a\hat b}, M_{\hat c\hat d}] &= -\frac{1}{2}(\eta_{\hat b\hat c}M_{\hat a\hat d} + \eta_{\hat a\hat d}M_{\hat b\hat c} - \eta_{\hat b\hat d}M_{\hat a\hat c} - \eta_{\hat a\hat c}M_{\hat b\hat d})\\
[T_r, T_s] &= -g\epsilon_{rst}T_t\\
[M_{\hat a\hat b}, Q_{A\alpha}] &= -\frac{1}{4}(\gamma_{\hat a\hat b})^{\beta}{}_{\alpha}Q_{A\beta}\\
[T_r, Q_{A\alpha}] &= \frac{i}{2}g\sigma_{rA}^{\ B}Q_{B\alpha}\\
\{Q_{A\alpha}, Q_{B\beta}\} &= -2i\sigma^r_{AB}C_{(-)\alpha\beta}T_r + \epsilon_{AB}(C_{(-)}\gamma^{\hat a\hat b})_{\alpha\beta}M_{\hat a\hat b}\,.
\end{aligned} \tag{2.1}$$

Here $M_{\hat a\hat b}$, $\hat a, \hat b = 0, \ldots, 6$ are the $\mathrm{SO}(t, 7-t)$ generators, preserving the metric

$$\eta_{\hat a\hat b} = (\underbrace{+\ldots+}_{t\text{ times}}\underbrace{-\ldots-}_{7-t\text{ times}})\,. \tag{2.2}$$

In the following we will keep the notation "t" for any of the vector timelike indices and we will denote "s" any of the $7-t$ vector spacelike indices of the metric.

In equations (2.1) T^r, $r = 1, 2, 3$ are the $\mathrm{SU}(2)$ generators, $\sigma^r{}_A{}^C$ are the usual Pauli matrices and we set $\sigma^r_{AB} = \sigma^r_{BA} \equiv \sigma^r{}_A{}^C\epsilon_{CB}$, where $\epsilon_{AB} = -\epsilon_{BA} = -\epsilon^{BA} = \epsilon^{AB}$ is the antisymmetric $\mathrm{SU}(2)$ tensor which can be used to raise and lower the $\mathrm{SU}(2)$ indices according to the following rules:

$$T^{\ldots A\ldots} = \epsilon^{AB}\ T^{\ldots}{}_B{}^{\ldots} \tag{2.3}$$

$$T_{\ldots A\ldots} = T_{\ldots}{}^B{}_{\ldots}\ \epsilon_{BA}\,. \tag{2.4}$$

Note that, since the bosonic subgroup is $\mathrm{SO}(t, s)$, $t + s = 7$, the spinor Clifford algebra is seven-dimensional. Indeed $Q_{A\alpha}$, $\alpha = 1, \ldots, 8$, $A = 1, 2$ are the odd generators of the superalgebra (supersymmetry charges) consisting of seven-dimensional 8-component symplectic-Majorana spinors, and $C_{(-)\alpha\beta}$ is the symmetric charge conjugation matrix in $D = 7$ satisfying $C^T_{(-)} = C_{(-)}$, $C^2_{(-)} = \mathbf{1}$. Note that the $D = 7$ gamma matrices are $C_{(-)}$-antisymmetric, that is they satisfy

$$\gamma^T_{\hat a} = -C^{-1}_{(-)}\gamma_{\hat a}C_{(-)}\,. \tag{2.5}$$

We note that the symplectic-Majorana condition holds for both cases $t = 1$, $t = 2$ since we have $\rho = s - t = 5$, $\rho = s - t = 3$ respectively, so that the symplectic-Majorana condition on the spinors can be imposed for both signatures [11],

$$t = 1\,, \qquad s = 6 \qquad \eta_{\hat{a}\hat{b}} = (+ - - - - - -) \tag{2.6}$$

$$t = 2\,, \qquad s = 5 \qquad \eta_{\hat{a}\hat{b}} = (+ + - - - - -)\,. \tag{2.7}$$

In fact the spinor charges $Q_{A\alpha}$ satisfy in both cases the symplectic-Majorana condition

$$\overline{(Q_A)} = \epsilon^{AB} {Q_B}^T C_{(-)}\,. \tag{2.8}$$

On the other hand, the definition of the Dirac conjugate spinor is given by

$$\overline{(Q_A)} = (Q_A)^\dagger G^{-1}\,. \tag{2.9}$$

We have two possible choices for the matrix G in $D = t + s$ dimensions [12]: the first choice corresponds to the product of all the timelike gamma matrices, that is:

$$G_I = \gamma^0 \cdots \gamma^{t-1} \tag{2.10}$$

while the second choice corresponds to take the product of all the spacelike gamma matrices:

$$G_{II} = \gamma^{D-s} \cdots \gamma^{D-1}\,. \tag{2.11}$$

Correspondingly the hermitean conjugate of the gamma matrices is given by:

$$\gamma^{a\dagger} = \eta\, G^{-1} \gamma^a G \tag{2.12}$$

where the phase η takes two different values according to the two choices of G, namely

$$\eta_I = (-1)^{t-1}\,; \qquad \eta_{II} = (-1)^s\,. \tag{2.13}$$

For seven dimensional spinors we choose to use G_I for both signatures. However the consideration of both conventions will be relevant when we will reduce the MCE to retrieve the six dimensional theory.

We note that the consistency of the last equation of (2.1) requires an appropriate definition of the hermitean conjugate of the anticommutator in order that both T_r and $M_{\hat{a}\hat{b}}$ be antihermitean generators for both signatures. It is convenient to discuss this point in terms of the dual formulation of the algebra (2.1). Introducing the $F^t(4)$ Lie algebra valued 1-forms $\omega^{\hat{a}\hat{b}}$, A^r, $\psi^{A\alpha}$ satisfying:

$$\omega^{\hat{a}\hat{b}}(M_{\hat{c}\hat{d}}) = \delta^{\hat{a}\hat{b}}_{\hat{c}\hat{d}}\,, \qquad A^r(T_s) = \delta^r_s\,, \qquad \psi^{A\alpha}(Q_{B\beta}) = \delta^A_B \delta^\alpha_\beta \tag{2.14}$$

and using the well known relation

$$d\mu^A(X_B, X_C) = -\frac{1}{2}\mu^A([X_B, X_C]) \tag{2.15}$$

where X_B, X_C are the generators of the super-Lie algebra, and μ^A are their dual forms, one easily obtains (omitting the wedge product symbol):

$$\begin{aligned} d\omega^{\hat{a}\hat{b}} - \omega^{\hat{a}}{}_{\hat{c}}\omega^{\hat{c}\hat{b}} + \frac{1}{2}\bar{\psi}_A\gamma^{\hat{a}\hat{b}}\psi^A &= 0 \\ d\psi_A - \frac{1}{4}\omega^{\hat{a}\hat{b}}\gamma_{\hat{a}\hat{b}}\psi_A + \frac{i}{2}g\sigma^r_{AB}A_r\psi^B &= 0 \\ dA^r - \frac{1}{2}g\epsilon^{rst}A_sA_t - i\sigma^r_{AB}\bar{\psi}^A\psi^B &= 0\,. \end{aligned} \tag{2.16}$$

Note that ψ_A satisfies the same properties as Q_A, namely it is a symplectic-Majorana spinor 1-form obeying

$$\overline{(\psi_A)} \equiv \psi^\dagger G^{-1} = \epsilon^{AB}\psi_B{}^T C_{(-)}\,. \tag{2.17}$$

When the algebra $F^t(4)$ is written in this form, one can easily check its closure under d-differentiation, which amounts to the closure of the Jacobi identities in equations (2.1). One finds that it closes if and only if $g = 3/2$. The verification relies on the following Fierz identity valid for symplectic-Majorana spinor 1-forms in $D = 7$:

$$\frac{1}{4}\gamma_{\hat{a}\hat{b}}\psi_A\bar{\psi}_B\gamma^{\hat{a}\hat{b}}\psi^B + 3\psi^B\bar{\psi}_A\psi_B = 0\,. \tag{2.18}$$

We now observe that in arbitrary dimension $D = t+s$, the reality of the currents $j^{a_1\cdots a_n} \equiv \bar{\psi}_A\gamma^{a_1\cdots a_n}\psi_B$ (where $\gamma^{a_1\cdots a_n}$ is a totally antisymmetric product of gamma matrices) depends on the values of t and s appearing in the metric η_{ab}. Indeed we have [12]

$$\begin{aligned} (j^{a_1\cdots a_n})^* &= -\beta\chi j^{a_1\cdots a_n} \\ \chi_I &= (-1)^{\frac{1}{2}(t-n-1)(t-n)} \\ \chi_{II} &= (-1)^{\frac{1}{2}(n-s-1)(n-s)} \end{aligned} \tag{2.19}$$

where the subscripts I and II refer to the two possible choices for the matrix $G = \{G_I,\ G_{II}\}$ and $\beta = \pm 1$ is the arbitrary phase appearing in the convention one uses for the definition of the complex conjugate of the product of two spinors, namely [3, 12]

$$(\lambda\mu)^* = \beta\lambda^*\mu^* = -\beta\mu^*\lambda^*\,. \tag{2.20}$$

One can verify that the two currents appearing in equations (2.16) behave under complex conjugation (using $G = G_I$) as shown in table 1.

	$\bar{\psi}_A\psi_B$	$\bar{\psi}_A\gamma^{\hat{a}}\psi_B$	$\bar{\psi}_A\gamma^{\hat{a}\hat{b}}\psi_B$
$(t,s) = (1,6)$	$\chi_I = +1$	$\chi_I = +1$	$\chi_I = -1$
$(t,s) = (2,5)$	$\chi_I = -1$	$\chi_I = +1$	$\chi_I = +1$

Table 1: Values of the phase χ_I for $D = 7$ currents

Consistency of equations (2.16) requires that $\bar{\psi}_A\gamma^{\hat{a}\hat{b}}\psi_B$ is real and $\bar{\psi}_A\psi_B$ is pure imaginary, so that, if we want to have the same algebra for both signatures, we have to choose $\beta = 1$ for $(t,s) = (1,6)$ and $\beta = -1$ for $(t,s) = (2,5)$. Alternatively one could use the same convention for β, say $\beta = 1$, in which case equations (2.16) would refer to the $(t,s) = (1,6)$ case, while the $(t,s) = (2,5)$ would give analogous equations with an i factor in front of the two currents. We choose the first alternative and the corresponding algebra of (anti)-commutators is the one written in equations (2.1) which indeed corresponds to both choices for $\eta_{\hat{a}\hat{b}}$.

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The foregoing discussion is propaedeutical to the search of *de Sitter* and *anti de Sitter* supersymmetric $D = 6$ backgrounds. To reach this goal we reduce the seven dimensional indices $\hat{a}$, $\hat{b} = 0, 1, \ldots, 6$ to six dimensional ones a, $b = 0, 1, \ldots, 5$ plus the index 6. Furthermore the reduced 1-forms can be now interpreted as living on the pull-back of $F^t(4)$ to the coset $F^t(4)/\,\mathrm{SO}(1,5) \otimes \mathrm{SU}(2) \supset \mathrm{SO}(t,s)/\,\mathrm{SO}(1,5)$ for both choices of t and s (2.6), (2.7).

In order to obtain such backgrounds in $D = 6$ we reduce the indices in such a way that the index "6" is of type "t" for the signature $(t,s) = (2,5)$, corresponding to an AdS background, while to obtain a dS background we start from the signature $(t,s) = (1,6)$ and perform the reduction in such a way that the index "6" is of type "s". Note that the procedure is consistent also for the odd part of the algebra, because the gamma matrices are the same in $D = 6$ and $D = 7$, and the spinors in $D = 6$ with Minkowski signature $(t,s) = (1,5)$, are still symplectic-Majorana since $\rho = s - t = 4$. Furthermore we can still use $C_{(-)}$ as charge conjugation matrix, since in six dimensions we have two possible charge conjugation matrices $C_{(+)}$, $C_{(-)}$ and we can choose $C_{(-)}$ coincident with the $C_{(-)}$ defined for the $D = 7$ spinor algebra. However, there is a difference between the two cases: reducing on a "s" type direction (dS_6 case) the number of timelike gamma matrices is unchanged, so we can keep the definition of the barred spinor in $D = 6$ as

$$\bar{\psi}_A = \psi_A^\dagger \gamma^0 \equiv \psi_A^\dagger G_I^{-1}\,. \tag{2.21}$$

On the other hand, if we reduce on a "t" direction (AdS_6 case), the seven dimensional definition $\bar{\psi}_A = \psi_A^\dagger(-\gamma^0\gamma^6)$ contains the γ^6 matrix that is no more associated to a space-time direction in $D = 6$. This is a crucial point, and the consequences can be understood considering that in the two cases we have:

$$\begin{array}{llll} AdS: & \eta_{66} = 1 \Longrightarrow \gamma_6 = \gamma^6 \equiv \gamma_6^{(+)}\,, & (\gamma_6^{(+)})^2 = 1\,, & (\gamma_6^{(+)})^\dagger = \gamma_6^{(+)} \\ dS: & \eta_{66} = -1 \Longrightarrow \gamma_6 = -\gamma^6 \equiv \gamma_6^{(-)}\,, & (\gamma_6^{(-)})^2 = -1\,, & (\gamma_6^{(-)})^\dagger = -\gamma_6^{(-)}\,. \end{array} \tag{2.22}$$

From a $D = 6$ point of view, the matrix γ_6 is usually called "γ_7" and is defined as $\gamma_7 = \alpha\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5$ with $|\alpha| = 1$. If we want to preserve the properties (2.22) we choose

$$\gamma_7 = \gamma_6^{(-)} = i\gamma_6^{(+)} = i\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5\,. \tag{2.23}$$

This means that starting from $D = 7$ with $(t,s) = (2,5)$, where

$$\bar{\psi}_A = \psi_A^\dagger(-\gamma^0\gamma^6_{(+)}) \tag{2.24}$$

and reducing on a "t" type direction, the Dirac conjugate spinor in $D = 6$ turns out to be defined as

$$\bar{\psi}_A = \psi_A^\dagger(-\gamma^0\gamma^6_{(+)}) = \psi_A^\dagger(i\gamma^0\gamma^7) = \psi_A^\dagger(-\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5) \equiv \psi_A^\dagger G_{II}^{-1} \tag{2.25}$$

that is, we must use convention II to define the Dirac conjugate spinor.

We can now perform the pull-back to the six dimensional superspace. Note that since we are doing the pull-back $F^t(4) \longrightarrow F^t(4)/\,\mathrm{SO}(1,5)$ the spin connection components ω^{a6}

assume the meaning of the vielbein cotangent frame on the coset. Introducing a (real) rescaling parameter m we set:

$$\begin{aligned}\omega^{a6} &\longrightarrow 2mV^a\\ \psi_A &\longrightarrow \sqrt{2m}\psi_A\\ A_r &\longrightarrow 2mA_r\\ g &\longrightarrow \frac{1}{2m}g\end{aligned}$$

so that the M.C.E. (2.16) reduce to the following form:

$$\mathcal{D}V^a + \frac{1}{2}\overline{\psi}_A\gamma^a\gamma^{6(\pm)}\psi^A = 0 \tag{2.26}$$

$$\mathcal{R}^{ab} + 4m^2\, V^aV^b\eta_{66} + m\overline{\psi}_A\gamma^{ab}\psi^A = 0 \tag{2.27}$$

$$dA^r - \frac{1}{2}\, g\,\epsilon^{rst}A_sA_t - i\,\overline{\psi}_A\psi_B\,\sigma^{rAB} = 0 \tag{2.28}$$

$$D\psi_A + m\gamma_a\gamma_6^{(\pm)}\psi_AV^a = 0 \tag{2.29}$$

where $\mathcal{R}^{ab}$ is the Lorentz curvature 2-form in $D = 6$, namely

$$\mathcal{R}^{ab} = d\omega^{ab} - \omega^a_{\ c}\omega^{cb} \tag{2.30}$$

and the SU(2) vector field strength is defined as

$$\mathcal{F}^r = dA^r - \frac{1}{2}\, g\,\epsilon^{rst}A_sA_t\,. \tag{2.31}$$

Moreover, we have defined the Lorentz covariant derivatives in $D = 6$ as follows

$$\mathcal{D}V^a = dV^a - \omega^{ab}V_b \tag{2.32}$$

$$\mathcal{D}\psi_A = d\psi_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}\psi_A \tag{2.33}$$

and the $\mathrm{SU}(2)\times\mathrm{SO}(1,5)$ covariant derivative on ψ_A as

$$D\psi_A = \mathcal{D}\psi_A + \frac{i}{2}\sigma^r_{AB}A_r\psi^B\,. \tag{2.34}$$

Equations (2.26)–(2.29) define a supersymmetric vacuum configuration of $D = 6$, $N = 2$ supergravity in terms of the superforms V^a, ω^{ab}, A_r, ψ_A.

If we now perform d-differentiation of equations (2.26)–(2.29), we find that they of course close, since they are merely a rewriting of equations (2.16); however the relevant Fierz identity to be used in this case, namely in $D = 6$ is the following

$$\frac{m}{4}\gamma_{ab}\psi_A\bar{\psi}_B\gamma^{ab}\psi^B + g\psi^B\bar{\psi}_A\psi_B + \frac{m}{2}\gamma_6\gamma_a\psi_A\bar{\psi}_B\gamma^6\gamma^a\psi^B = 0 \tag{2.35}$$

which only holds when $g = 3m$. Indeed in this case equation (2.35) reduces to equation (2.18) written in six dimensional formalism.

Note that the dependence of this equations from the signature (t, s) is hidden in the different values of η_{66} and properties of $\gamma_6^{(\pm)}$ as given in equations (2.22).

In particular, we see that, restricting the forms to space-time, so that $\psi_{A\mu} = 0$, equation (2.27) gives

$$\mathcal{R}^{ab}_{cd} = -4m^2\eta_{66}\delta^{ab}_{cd} \longrightarrow \mathcal{R}_{\mu\nu} = 20\eta_{66}m^2 g_{\mu\nu} \tag{2.36}$$

so that, depending on the signature $(t, s) = (2, 5)$ or $(t, s) = (1, 6)$, we get a supersymmetric *anti de Sitter* or *de Sitter* background with cosmological constant $\pm 20m^2$ respectively. As we stressed in the introduction, the fact that we can obtain a supersymmetric *de Sitter* configuration is a peculiar property of the six dimensional theory based on the $F(4)$ supergroup which admits two real sections, one with bosonic subgroup $\mathrm{SO}(2, 5) \times \mathrm{SU}(2)$ and the other with bosonic subgroup $\mathrm{SO}(1, 6) \times \mathrm{SU}(2)$.

On the other hand, the supersymmetric configuration described before, is not complete, since the $D = 6$ $F(4)$ supergravity theory contains, besides the previously treated degrees of freedom, also an additional vector A_μ and a 2-form $B_{\mu\nu}$. The way to introduce these extra fields in the configuration is well known [6, 7] and consists in enlarging the MCE (2.26)–(2.29) into a Free Differential Algebra (FDA). Indeed let us add the following 2-form and 3-form equations to the MCE (2.26)–(2.29):

$$dA - mB + \alpha\,\overline{\psi}_A\gamma_6^{(\pm)}\psi^A = 0 \tag{2.37}$$

$$dB + \beta\,\overline{\psi}_A\gamma_a\psi^A V^a = 0 \tag{2.38}$$

where α and β are suitable constants. Performing d-differentiation of equations (2.37)–(2.38), using the Fierz identity

$$\overline{\psi}_A\gamma_a\psi^A\overline{\psi}_B\gamma^6\gamma^a\psi^B = 0 \tag{2.39}$$

the MCE (2.16) and the fact that and $\gamma_{ab}\gamma_6$ is $C_{(-)}$ symmetric, one easily finds

$$\beta = 2\alpha\eta_{66}\,. \tag{2.40}$$

From table 1 we see that in signature $(t, s) = (2, 5)$ the vector current has the same reality as the tensor current, therefore it is real; vice versa in signature $(t, s) = (1, 6)$ they have opposite reality, that is the vector current is pure imaginary. Hence we choose $\beta = 2$ and $\beta = 2i$ for $(t, s) = (2, 5)$ or $(t, s) = (1, 6)$ respectively, to respect the reality of the fields A and B.

At this point we have all the necessary ingredients to construct the six dimensional supergravity theories which will admit as supersymmetric background either a six dimensional *anti de Sitter* ($(t, s) = (2, 5)$) or a *de Sitter* configuration ($(t, s) = (1, 6)$). Hereafter we will refer to these two theories as AdS_6 or dS_6 supergravity respectively.

Actually AdS_6, $N = 2$, $F(4)$ supergravity which was constructed by Romans [5], by means of the Noether approach, using the supergravity multiplet only was reformulated in terms of a FDA in superspace in references [6, 7]. Furthermore its coupling to matter multiplets and the gauging of a compact group was given. However, the starting FDA used in [6, 7] was defined with a different representation for the gamma matrices. We can map our FDA (2.26)–(2.29), (2.37)–(2.38) into that of [6, 7] by the following redefinition which preserves the square of the gamma matrices (and hence the metric $(t, s) = (2, 5)$):

$$\gamma^a \longrightarrow -i\gamma^a\,\gamma^{6(+)} \equiv \gamma^7\,\gamma^a\,. \tag{2.41}$$

With the previous redefinition, the Dirac conjugate spinor in six dimensions, which was previously defined using convention II (see equation (2.25)) must be now defined using convention I. In fact, for seven dimensional spinors with $(t,s)=(2,5)$ we had $\psi^\dagger G_I^{-1} \equiv \psi^\dagger(-\gamma^0\gamma^{6(+)})$. If we now go to six dimensions, the spinors do not reduce, and performing the redefinition (2.41) we obtain

$$\psi^\dagger(-\gamma^0\gamma^{6(+)}) \longrightarrow \psi^\dagger(-\gamma_7\gamma^0\gamma^{6(+)}) = i\psi^\dagger\gamma^0 \equiv i\psi^\dagger G_I^{-1} \tag{2.42}$$

and the AdS_6 FDA takes the following form:

$$\mathcal{D}V^a - \frac{i}{2}\overline{\psi}_A\gamma^a\psi^A = 0 \tag{2.43}$$
$$\mathcal{R}^{ab} + 4m^2\, V^aV^b + m\overline{\psi}_A\gamma^{ab}\psi^A = 0 \tag{2.44}$$
$$dA^r + \frac{1}{2}g\,\epsilon^{rst}A_sA_t - i\,\overline{\psi}_A\psi_B\,\sigma^{rAB} = 0 \tag{2.45}$$
$$D\psi_A - im\gamma_a\psi_AV^a = 0 \tag{2.46}$$
$$dA - mB - i\,\overline{\psi}_A\gamma_7\psi^A = 0 \tag{2.47}$$
$$dB + 2\,\overline{\psi}_A\gamma_7\gamma_a\psi^AV^a = 0\,. \tag{2.48}$$

The corresponding supergravity has been thoroughly discussed in [6, 7] and we do not dwell on it anymore.

Our interest instead is to construct the *de Sitter* supergravity and in order to profit of the results obtained in the AdS_6 case, it is convenient to redefine also for dS_6 supergravity the gamma matrix representation as follows:

$$\gamma^a \longrightarrow -\gamma^a\,\gamma^{6(-)} \equiv \gamma^7\,\gamma^a\,. \tag{2.49}$$

With the previous redefinition, the Dirac conjugate spinor in six dimensions must be defined using convention II, that is with opposite convention with respect to equation (2.21). Indeed, for seven dimensional spinors with $(t,s)=(1,6)$ we have $\psi^\dagger G_I^{-1} \equiv \psi^\dagger\gamma^0$. Proceeding as before and using now (2.49) we obtain

$$\psi^\dagger\gamma^0 \longrightarrow \psi^\dagger\gamma_7\gamma^0 = \psi^\dagger(i\gamma^0\gamma^1\gamma^2\gamma^3\gamma^4\gamma^5)\gamma^0 \equiv i\psi^\dagger G_{II}^{-1}\,. \tag{2.50}$$

The resulting FDA for dS_6 supergravity is therefore

$$\mathcal{D}V^a + \frac{1}{2}\overline{\psi}_A\gamma^a\psi^A = 0 \tag{2.51}$$
$$\mathcal{R}^{ab} - 4m^2\, V^aV^b + m\overline{\psi}_A\gamma^{ab}\psi^A = 0 \tag{2.52}$$
$$dA^r + \frac{1}{2}g\,\epsilon^{rst}A_sA_t - i\,\overline{\psi}_A\psi_B\,\sigma^{rAB} = 0 \tag{2.53}$$
$$D\psi_A - m\gamma_a\psi_AV^a = 0 \tag{2.54}$$
$$dA - mB - i\,\overline{\psi}_A\gamma_7\psi^A = 0 \tag{2.55}$$
$$dB - 2i\,\overline{\psi}_A\gamma_7\gamma_a\psi^AV^a = 0\,. \tag{2.56}$$

As a check of our computation we can verify that all the terms involving bilinear currents in equations (2.43)–(2.48), (2.51)–(2.56) are real, consistently with the fact that

	$\bar\psi_a\psi_B$	$\bar\psi_a\gamma_a\psi_B$	$\bar\psi_a\gamma_{ab}\psi_B$	$\bar\psi_a\gamma_7\gamma_a\psi_B$	$\bar\psi_a\gamma_7\psi_B$
I	$\chi_I=-1$	$\chi_I=-1$	$\chi_I=+1$	$\chi_I=+1$	$\chi_I=-1$
II	$\chi_{II}=+1$	$\chi_{II}=-1$	$\chi_{II}=-1$	$\chi_{II}=+1$	$\chi_{II}=+1$

Table 2: Values of the phases χ_I, χ_{II} for $D=6$ currents.

the bosonic fields must be real. Indeed, taking into account that the Dirac conjugate of the spinors in both cases have an extra i factor with respect to the usual definition (2.42), (2.50), that the γ_7 matrix has an explicit i factor (2.23) and using furthermore the relation

$$\gamma_7\gamma_{a_1\dots a_p}=\frac{i}{(D-p)!}\epsilon_{a_1\dots a_p\,b_{p+1}\dots b_D}\gamma^{b_{p+1}\dots b_D} \tag{2.57}$$

one obtains the phases shown in table 2 for the relevant six dimensional currents

We can see that if we set, consistently with the choice in $D=7$, $\beta=1$ for the dS_6 case and $\beta=-1$ for the AdS_6 one, all the terms containing the bilinear currents in equations (2.43)–(2.48), (2.51)–(2.56) are real, implying the reality of the corresponding bosonic terms.

A further check is given by the comparison of equations (2.46) and (2.54): indeed these two equations can be interpreted as the vanishing of the AdS_6 and dS_6 covariant derivatives. They differ by a factor $-i$ in the terms proportional to m and one can wonder whether they are both consistent with the symplectic-Majorana condition (2.8). The answer is yes; indeed, let us consider a covariant derivative ∇ defined with a parameter q, where $q=im$ for AdS_6 and $q=m$ for dS_6:

$$\nabla\psi_A=D\psi_A-q\gamma_a\psi_AV^a \tag{2.58}$$

and let us apply the symplectic-Majorana condition (2.17) to the spinor $\nabla\psi_A$; using equations (2.5) and (2.12), we have:

$$\begin{aligned}\overline{\nabla\psi_A}&=D\psi_A^\dagger G^{-1}-q^*\psi_A^\dagger G^{-1}G\gamma_a^\dagger G^{-1}V^a=D\overline{\psi}_A-q^*\eta\,\overline{\psi}_A\gamma_aV^a\\ \epsilon^{AB}\nabla\psi_B^TC_{(-)}&=\epsilon^{AB}D\psi_B^TC_{(-)}-q\epsilon^{AB}\psi_B^TC_{(-)}C_{(-)}^{-1}\gamma_a^TC_{(-)}V^a=D\bar\psi_A+q\bar\psi_A\gamma_aV^a\,.\end{aligned}$$

Equating the two r.h.s., and using equation (2.13) we see that the only two consistent choices are: $q=im$ using convention I, that is in the AdS_6 case, and $q=m$ using convention II, that is in the dS_6 case.

3. $D=6$ *de Sitter* supergravity: transformation rules and lagrangian

In this section we give the form of the supersymmetry transformation laws and the lagrangian of dS_6 supergravity.

Since the underlying algebra is quite analogous to the AdS_6 algebra, it is simple to transform the result of the AdS_6 theory into the present case. Indeed it is easy to verify (see appendix B) that formally we can map the AdS_6 FDA (2.43)–(2.48) into the dS_6

FDA (2.51)–(2.56) by the following substitutions:

$$\begin{aligned} m &\longrightarrow -im \\ g &\longrightarrow -ig \\ A^r &\longrightarrow iA^r \\ A &\longrightarrow iA \\ B &\longrightarrow -B \\ \psi_A &\longrightarrow \psi_A \\ \bar{\psi}_A &\longrightarrow i\bar{\psi}_A\,. \end{aligned} \tag{3.1}$$

Out of the vacuum we must also perform the following substitutions to get the correct form of the superspace curvatures and hence of the supersymmetry transformation laws:

$$\begin{aligned} \sigma &\longrightarrow \sigma \\ \chi_A &\longrightarrow -i\chi_A \\ \overline{\chi}_A &\longrightarrow i\overline{\chi}_A \\ \lambda^I_A &\longrightarrow -i\lambda^I_A \\ \overline{\lambda}^I_A &\longrightarrow i\overline{\lambda}^I_A \\ A^I &\longrightarrow iA^I \end{aligned} \tag{3.2}$$

where χ_A is a spin 1/2 field and σ a scalar field belonging to the graviton multiplet, A^I, λ^I_A, P^I_α ($\alpha = 1, 2, 3, 4$, $I = 1, \dots, n$) are the vectors, spin 1/2 and scalars of the n vector multiplets (the scalars are described by the vielbein of the coset $\mathrm{SO}(4,n)/\,\mathrm{SO}(4) \otimes \mathrm{SO}(n)$). These substitutions, besides reproducing the dS_6 FDA (2.51)–(2.56), and the supersymmetry transformation laws out of the vacuum given below, leave unchanged the gauge invariance

$$B \longrightarrow B + d\lambda\,, \qquad A \longrightarrow A + m\lambda \tag{3.3}$$

the R-symmetry and the Fierz identity (2.35).

Using the map (3.1) we can recover the dS_6 supersymmetry transformation laws and lagrangian by applying it to the corresponding transformation laws and lagrangian of AdS_6 theory. We have checked that the resulting theory coincides with the one obtained performing explicit calculation of the Bianchi identities and of the lagrangian in the geometric approach. Note that, equation (3.1), (3.2) tells us that the kinetic terms of the vectors undergo a changes of sign with respect to the AdS_6 theory, implying that they behave as ghosts and also the cosmological constant undergoes a change of sign. This was to be expected since if one applies the Noether method to construct the $D = 6$ $F(4)$ theory as in reference [5] the resulting theory, with positive definite vector kinetic terms, is unique and of AdS_6 type.

We note however that, differently from what happens to variant supergravities discussed in the literature [13] the R-symmetry group of our dS_6 supergravity remains SU(2), that is it remains compact compared to the conventional AdS_6 supergravity. This is consistent with the fact that all the three vectors A^r in the adjoint of SU(2) have become ghosts in the dS_6 theory.

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For the sake of clarity we begin to give the supersymmetry transformation laws for the theory without matter coupling, where all the important changes with respect to the AdS_6 theory already take place. We have

$$
\begin{aligned}
\delta V^a_\mu &= \overline{\psi}_{A\mu}\gamma^a\varepsilon^A && (3.4)\\
\delta B_{\mu\nu} &= -4i\epsilon^{-2\sigma}\overline{\chi}_A\gamma_7\gamma_{\mu\nu}\varepsilon^A + 4ie^{-2\sigma}\overline{\varepsilon}_A\gamma_7\gamma_{[\mu}\psi^A_{\nu]} && (3.5)\\
\delta A_\mu &= -2ie^{\sigma}\overline{\chi}_A\gamma_7\gamma_\mu\varepsilon^A + 2ie^{\sigma}\overline{\varepsilon}_A\gamma^7\psi^A_\mu && (3.6)\\
\delta A^r_\mu &= -2ie^{\sigma}\overline{\chi}^A\gamma_\mu\varepsilon^B\sigma^r_{AB} + 2ie^{\sigma}\sigma^{rAB}\overline{\varepsilon}_A\psi_{B\mu} && (3.7)\\
\delta\psi_{A\mu} &= D_\mu\varepsilon_A + \frac{i}{16}e^{-\sigma}[\epsilon_{AB}F_{\nu\lambda}\gamma_7 - \sigma_{rAB}F^r_{\nu\lambda}](\gamma_\mu^{\nu\lambda} - 6\delta^\nu_\mu\gamma^\lambda)\varepsilon^B - \\
&\quad - \frac{i}{32}e^{2\sigma}H_{\nu\lambda\sigma}\gamma_7(\gamma_\mu{}^{\nu\lambda\sigma} - 3\delta^\nu_\mu\gamma^{\lambda\sigma})\varepsilon_A - \frac{1}{4}g\,e^{\sigma}\gamma_\mu\varepsilon_A - \frac{1}{4}me^{-3\sigma}\gamma_\mu\varepsilon_A\\
\delta\chi_A &= -\frac{1}{2}\gamma^\mu\partial_\mu\sigma\varepsilon_A - \frac{i}{16}e^{\sigma}[\epsilon_{AB}F_{\mu\nu}\gamma_7 + \sigma_{rAB}F^r_{\mu\nu}]\gamma^{\mu\nu}\varepsilon^B - \\
&\quad - \frac{i}{32}e^{2\sigma}H_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A - \frac{1}{4}ge^{\sigma}\varepsilon_A + \frac{3}{4}me^{-3\sigma}\varepsilon_A && (3.8)\\
\delta\sigma &= \overline{\chi}_A\varepsilon^A\,. && (3.9)
\end{aligned}
$$

From the transformation laws (3.4)–(3.9), it is easy to see that one can obtain a *deSitter* supersymmetric background choosing $g = 3m$. (Recall that in the vacuum, besides putting to zero the field-strengths, we also set $\sigma = 0$). In this way we obtain:

$$
\begin{aligned}
\delta\chi_A &\equiv -\frac{1}{4}(g - 3m)\,\varepsilon_A = 0 && (3.10)\\
\delta\psi_{A\mu} &\equiv D_\mu\varepsilon_A - \left(\frac{1}{4}m + \frac{1}{4}g\right)\gamma_\mu\varepsilon_A = D_\mu\varepsilon_A - m\gamma_\mu\varepsilon_A = \nabla^{dS}_\mu\epsilon_A && (3.11)\\
R^{ab} &\equiv -\frac{1}{2}R^{ab}_{cd}V^cV^d = 4m^2V^aV^b \to R_{\mu\nu} = -20m^2g_{\mu\nu} && (3.12)\\
&\big(\mathcal{F}^r_{\mu\nu} = \mathcal{F}_{\mu\nu} - mB_{\mu\nu} = H_{\mu\nu\rho} = \chi_A = \psi_{A\mu} = \sigma = 0\big) && (3.13)
\end{aligned}
$$

which corresponds to a dS configuration with dS radius $R^2_{dS} = (4m^2)^{-1}$

$$
\begin{aligned}
(detV)^{-1}\mathcal{L} = &-\frac{1}{4}\mathcal{R} + \frac{1}{8}e^{-2\sigma}\left[\widehat{\mathcal{F}}_{\mu\nu}\widehat{\mathcal{F}}^{\mu\nu} + \mathcal{F}^r_{\mu\nu}\mathcal{F}^{\mu\nu}_r\right] + \frac{3}{64}e^{4\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho} - \\
&- \frac{1}{2}\overline{\psi}_{A\mu}\gamma^{\mu\nu\rho}\nabla_\nu\psi^A_\rho + 2\overline{\chi}_A\gamma^\mu\nabla_\mu\chi^A + \partial^\mu\sigma\partial_\mu\sigma - \\
&- 2\left(-\frac{1}{4}ge^{\sigma} - \frac{1}{4}me^{-3\sigma}\right)\overline{\psi}_{\mu A}\gamma^{\mu\nu}\psi^A_\nu + 4\left(-\frac{1}{4}ge^{\sigma} + \frac{3}{4}me^{-3\sigma}\right)\overline{\psi}_{\mu A}\gamma^\mu\chi^A - \\
&- \mathcal{W}^{ds}(\sigma;\, g, m) \qquad (3.14)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{W}^{ds} &= g^2e^{2\sigma} + 4mge^{-2\sigma} - m^2e^{-6\sigma}\\
\widehat{\mathcal{F}}_{\mu\nu} &\equiv \mathcal{F}_{\mu\nu} - mB_{\mu\nu}\,.
\end{aligned}
$$

We note the “wrong” sign in the vector kinetic term is in agreement with our previous discussion. If we count the number of bosonic ghosts and non ghosts degrees of freedom, we find that they are $16 + 16$ in agreement with the considerations of reference [14].

We further observe that $\mathcal{W}^{dS} = -\mathcal{W}^{AdS}$ so that $\mathcal{W}^{dS}$ has the same critical points of $\mathcal{W}^{AdS}$ for $g = 3m$. However, while in the AdS_6 case the critical point is a maximum, the dilaton mass is negative (but satisfies the Breitenlhoner-Friedman bound), in our case instead we have a minimum of the potential and positive mass for the dilaton ($m_\sigma^2 = 24m^2$ or $m_\sigma^2 = 6$ in *de Sitter* radius units). We also note that since the vacuum is supersymmetric, the corresponding extremum is stable. The above considerations can be straightforwardly extended to the *de Sitter* supergravity coupled to an arbitrary number of vector multiplets; one expects that in this case also the vectors of the vector multiplets become ghosts and again the scalar potential has a reversed sign with respect to the AdS_6 case. This is in fact what happens and the complete supersymmetry transformation rules and the lagrangian are given in the appendix A.

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A. The gauged matter coupled theory

In this appendix we will briefly discuss the structure of the gauged matter coupled theory; for a more detailed explanation on the complete procedure see [6, 7].

The only kind of supersymmetric matter in $D = 6$, $N = 2$ is given by the vector multiplets, which contain the following fields:

$$(A_\mu,\, \lambda_A,\, \phi^\alpha)^I \tag{A.1}$$

where $\alpha = 0, 1, 2, 3$ and the index I labels an arbitrary number n of such multiplets.

With $F^I_{\mu\nu}$ we will indicate the field strengths associated to the vector fields A^I_μ; since the duality group is $G = \mathrm{SO}(4, n) \times O(1, 1)$, they can be arranged with the graviton multiplet field strength $F_{\mu\nu}$, $F^r_{\mu\nu}$ into an unique $\mathrm{SO}(4, n)$ vector $F^\Lambda_{\mu\nu}$ with $\Lambda = 0, 1, 2, 3, \ldots, 3 + n$, transforming in the fundamental representation of $\mathrm{SO}(4, n)$.

The $4n$ scalars parametrize the coset manifold $\mathrm{SO}(4, n)/\,\mathrm{SO}(4) \times \mathrm{SO}(n)$; they appear in the supersymmetry transformation rules by means of the coset representative $L^\Lambda{}_\Sigma$ of the matter coset manifold, where $\Lambda, \Sigma, \ldots = 0, \ldots, 3+n$; decomposing the $\mathrm{SO}(4, n)$ indices with respect to $H = \mathrm{SO}(4) \times \mathrm{SO}(n)$ we have:

$$L^\Lambda{}_\Sigma = (L^\Lambda{}_\alpha, L^\Lambda{}_I) \tag{A.2}$$

where $\alpha = 0, 1, 2, 3$ and $I = 4, \ldots, 3 + n$. Furthermore, since we are going to gauge $\mathrm{SU}(2) \otimes \mathcal{G} \subset \mathrm{SO}(4) \times \mathrm{SO}(n)$, where $\mathcal{G}$ is an n-dimensional subgroup of $\mathrm{SO}(n)$ and $\mathrm{SU}(2)$ is the diagonal subgroup of $\mathrm{SO}(4)$ as in pure supergravity, we will also decompose $L^\Lambda{}_\alpha$ as

$$L^\Lambda{}_\alpha = (L^\Lambda{}_0, L^\Lambda{}_r)\,, \qquad \text{with } r = 1, 2, 3\,. \tag{A.3}$$

The cotangent frame on the scalar coset manifold is given by the vielbein $\widehat{P}^I{}_0$, $\widehat{P}^I{}_r$. They are the (I,α) components of the gauged left invariant 1-form $L^{-1}\nabla L$

$$\begin{aligned}\widehat{P}^I{}_0 &= \left(L^{-1}\right)^I{}_\Lambda \nabla L^\Lambda{}_0 \\ \widehat{P}^I{}_r &= \left(L^{-1}\right)^I{}_\Lambda \nabla L^\Lambda{}_r \,.\end{aligned} \tag{A.4}$$

where the covariant derivative is defined as

$$\nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma \tag{A.5}$$

where $f^\Lambda{}_{\Pi\Gamma}$ are the structure constants of $\mathrm{SU}(2)\otimes\mathcal{G}$.

For simplicity of notation, it is useful to introduce the "dressed" non abelian vector field strengths:

$$\hat{T}_{[AB]\mu\nu} \equiv \epsilon_{AB} L^{-1}_{0\Lambda}\left(F^\Lambda_{\mu\nu} - mB_{\mu\nu}\delta_{\Lambda 0}\right) \tag{A.6}$$

$$T_{(AB)\mu\nu} \equiv \sigma^r_{AB} L^{-1}_{r\Lambda} F^\Lambda_{\mu\nu} \tag{A.7}$$

$$T_{I\mu\nu} \equiv L^{-1}_{I\Lambda} F^\Lambda_{\mu\nu} \,. \tag{A.8}$$

The fermionic supersymmetry transformation rules of the gauged theory, contain shift terms linear in the parameters g, g' and m (respectively, the gauge coupling constants of $\mathrm{SU}(2)$ and $\mathcal{G}$ and (one half) the inverse of the dS_6 radius); they have been computed for the gauged matter coupled AdS_6 $F(4)$ theory in references [6, 7], to which we refer the interest reader for notations and definitions. The fermionic gauge shifts were denoted as follows:

$$\delta\psi_{A\mu} = \ldots S^{(g,g',m)}_{AB}\gamma_\mu\varepsilon^B \tag{A.9}$$

$$\delta\chi_A = \ldots N^{(g,g',m)}_{AB}\varepsilon^B \tag{A.10}$$

$$\delta\lambda^I_A = \ldots M^{I\ (g,g',m)}_{AB}\varepsilon^B \,. \tag{A.11}$$

We find that the matrices $N^{(g,g',m)}_{AB}$ and $M^{I\ (g,g',m)}_{AB}$, in the present dS_6 case, are exactly the same as before while the gravitino shift acquires a factor $-i$, so that we define a new matrix $\Sigma^{(g,g',m)}_{AB}$ which is given in terms of the old one by

$$\Sigma^{(g,g',m)}_{AB} = -iS^{(g,g',m)}_{AB} \,. \tag{A.12}$$

The resulting supersymmetry transformation rules for the gauged matter coupled $F(4)$ dS_6 supergravity are therefore:

$$\begin{aligned}
\delta V^a_\mu &= \overline{\psi}_{A\mu}\gamma^a\varepsilon^A \\
\delta B_{\mu\nu} &= -4ie^{-2\sigma}\overline{\chi}_A\gamma_7\gamma_{\mu\nu}\varepsilon^A + i4e^{-2\sigma}\overline{\varepsilon}_A\gamma_7\gamma_{[\mu}\psi^A_{\nu]} \\
\delta A^\Lambda_\mu &= -2ie^\sigma\overline{\varepsilon}^A\gamma_7\gamma_\mu\chi^B L^\Lambda_0\epsilon_{AB} - 2ie^\sigma\overline{\varepsilon}^A\gamma_\mu\chi^B L^{\Lambda r}\sigma_{rAB} + ie^\sigma L_{\Lambda I}\overline{\varepsilon}^A\gamma_\mu\lambda^{IB}\epsilon_{AB} + \\
&\quad + 2ie^\sigma L^\Lambda_0\overline{\varepsilon}_A\gamma^7\psi_B\epsilon^{AB} + 2ie^\sigma L^{\Lambda r}\sigma^{AB}_r\overline{\varepsilon}_A\psi_B \\
\delta\psi_{A\mu} &= \mathcal{D}_\mu\varepsilon_A + \frac{i}{16}e^{-\sigma}[\hat{T}_{[AB]\nu\lambda}\gamma_7 - T_{(AB)\nu\lambda}](\gamma_\mu{}^{\nu\lambda} - 6\delta^\nu_\mu\gamma^\lambda)\varepsilon^B -
\end{aligned}$$

$$
\begin{aligned}
& - \frac{i}{32} e^{2\sigma} H_{\nu\lambda\rho}\gamma_7(\gamma_\mu{}^{\nu\lambda\rho} - 3\delta^\nu_\mu \gamma^{\lambda\rho})\varepsilon_A + \Sigma^{(g,g',m)}_{AB}\gamma_\mu \varepsilon^B + \\
& + \frac{1}{2}\varepsilon_A \overline{\chi}^C \psi_{C\mu} + \frac{1}{2}\gamma_7 \varepsilon_A \overline{\chi}^C \gamma^7 \psi_{C\mu} - \gamma_\nu \varepsilon_A \overline{\chi}^C \gamma^\nu \psi_{C\mu} + \gamma_7\gamma_\nu \varepsilon_A \overline{\chi}^C \gamma^7 \gamma^\nu \psi_{C\mu} - \\
& - \frac{1}{4}\gamma_{\nu\lambda}\varepsilon_A \overline{\chi}^C \gamma^{\nu\lambda}\psi_{C\mu} - \frac{1}{4}\gamma_7\gamma_{\nu\lambda}\varepsilon_A \overline{\chi}^C \gamma^7 \gamma^{\nu\lambda}\psi_{C\mu} \\
\delta\chi_A &= -\frac{1}{2}\gamma^\mu \partial_\mu \sigma \varepsilon_A - \frac{i}{16} e^{-\sigma}\left[\hat{T}_{[AB]\mu\nu}\gamma_7 + T_{(AB)\mu\nu}\right]\gamma^{\mu\nu}\varepsilon^B - \frac{i}{32} e^{2\sigma} H_{\mu\nu\lambda}\gamma_7\gamma^{\mu\nu\lambda}\varepsilon_A + \\
& + N^{(g,g',m)}_{AB}\epsilon^B \\
\delta\sigma &= \overline{\chi}_A \varepsilon^A \\
\delta\lambda^I_A &= -\hat{P}^I_{ri}\sigma^r{}_{AB}\partial_\mu \phi^i \gamma^\mu \varepsilon^B + \hat{P}^I_{0i}\epsilon_{AB}\partial_\mu\phi^i\gamma^7\gamma^\mu\varepsilon^B - \frac{i}{2} e^{-\sigma} T^I_{\mu\nu}\gamma^{\mu\nu}\varepsilon_A + M^{I(g,g',m)}_{AB}\varepsilon^B \\
\hat{P}^I_{0i}\delta\phi^i &= \frac{1}{2}\overline{\lambda}^I_A \gamma_7 \varepsilon^A \\
\hat{P}^I_{ri}\delta\phi^i &= \frac{1}{2}\overline{\lambda}^I_A \varepsilon_B \sigma_r^{AB}\,.
\end{aligned} \tag{A.13}
$$

The complete supersymmetric gauged lagrangian can be obtained with a standard procedure using equations (A.13) or by using the map (3.1), (3.2) to the corresponding AdS_6 lagrangian given in reference [7]. We limit ourselves to write down the most interesting terms. Our result is:

$$
\begin{aligned}
\mathcal{L}_{\text{kin}} &= -\frac{1}{4}\mathcal{R} + \frac{1}{8} e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}\widehat{\mathcal{F}}^\Lambda_{\mu\nu}\widehat{\mathcal{F}}^{\Sigma\mu\nu} + \frac{3}{64} e^{4\sigma} H_{\mu\nu\rho}H^{\mu\nu\rho} - \\
& - \frac{1}{2}\overline{\psi}_{A\mu}\gamma^{\mu\nu\rho}\nabla_\nu \psi^A_\rho + 2\overline{\chi}_A \gamma^\mu \nabla_\mu \chi^A - \frac{1}{8}\overline{\lambda}^I_A \gamma^\mu \nabla_\mu \lambda^A_I + \\
& + \partial^\mu\sigma\partial_\mu\sigma - \frac{1}{4}\left(\widehat{P}^{I0}_i \widehat{P}_{I0j} + \widehat{P}^{Ir}_i \widehat{P}_{Ir,j}\right)\partial^\mu\phi^i\partial_\mu\phi^j\,;
\end{aligned} \tag{A.14}
$$

$$
\mathcal{L}_{\text{Chern-Simons}} = -\frac{1}{64}\epsilon^{\mu\nu\rho\sigma\lambda\tau}B_{\mu\nu}\left(\eta_{\Lambda\Sigma}\hat{\mathcal{F}}^\Lambda_{\rho\sigma}\hat{\mathcal{F}}^\Sigma_{\lambda\tau} + mB_{\rho\sigma}\hat{\mathcal{F}}^0_{\lambda\tau} + \frac{1}{3}m^2 B_{\rho\sigma}B_{\lambda\tau}\right) \tag{A.15}
$$

$$
\begin{aligned}
\mathcal{L}_{\text{gauging}} &= -2\overline{\psi}^A_\mu \gamma^{\mu\nu}\overline{\Sigma}_{AB}\psi^B_\nu + 4\overline{\psi}^A_\mu\gamma^\mu \overline{N}_{AB}\chi^B - \frac{1}{4}\overline{\psi}^A_\mu \gamma^\mu \overline{M}^I_{AB}\lambda^B_I - \\
& - \mathcal{W}^{ds}(\sigma,\, \phi^i; g, g', m)\,.
\end{aligned} \tag{A.16}
$$

The fermionic gauge shifts and the scalar potential of $\mathcal{L}_{\text{gauging}}$ appearing in equation (A.16) have been given in references [6, 7]. We note that, as it was to be expected from the covariance of the vectors under duality group $\mathrm{SO}(4,n)\otimes O(1,1)$, also the matter multiplet vector fields become ghosts.

Moreover, also for the matter coupled theory, the potential $\mathcal{W}^{ds}$ has the opposite sign with respect to the AdS_6 case, $\mathcal{W}^{ds} = -\mathcal{W}^{Ads}$. The dilaton and the matter scalar fields are not ghosts, but they all have positive mass, corresponding to a minimum of the potential. The linearized equations of motion for the scalar fields are:

$$
\Box\sigma + 24m^2\sigma = 0 \tag{A.17}
$$

$$
\Box q^{I0} + 16m^2 q^{I0} = 0 \tag{A.18}
$$

$$
\Box q^{Ir} + 24m^2 q^{Ir} = 0\,. \tag{A.19}
$$

If we use as mass unity the inverse dS radius, which in our conventions is $R_{dS}^{-2}{=}4m^2$ we get:

$$\begin{aligned} m^2_\sigma &= 6 \\ m^2_{q^{I0}} &= 4 \\ m^2_{q^{Ir}} &= 6 \end{aligned} \tag{A.20}$$

B. An alternative approach to the construction of dS_6 $F(4)$ superalgebra

In this appendix we give a detailed discussion and justification with regard to the derivation of the map (3.1), (3.2) which have been used in the main text.

Let us consider the $F(4)$ AdS_6 FDA (2.43)–(2.48); in order to obtain from it a dS_6 supersymmetric FDA, the most important point is of course to change the sign in front of $4m^2$ in (2.44) and to try to adjust the coefficients in the various terms of the other equations by imposing the d-closure of the new FDA. On the other hand, in the approach given in the main body of the paper, we have seen that the AdS_6 and the dS_6 background can be obtained reducing respectively on an "t" type direction or on an "s" direction, which implies a different definitions of the Dirac conjugate spinor (2.25), (2.21) for the two theories. Note that this is a general feature of D-dimensional theories with the same background signature $D = t + s$ that are obtained reducing a $D + 1$ supergravity on a "t" type direction, $D + 1 = (t + 1) + s$, or on a "s" type direction, $D + 1 = t + (s + 1)$.

To recover the dS_6 theory we can now proceed as follows. Consider the FDA (2.26)–(2.29), (2.37), (2.38) for the AdS_6 case, where the Dirac conjugate spinors are defined according to convention I.[1] If we now change convention I into convention II for the Dirac conjugate spinors (according to the reduction on a "s" type direction), the algebra is formally the same and closes as well for $g = 3m$, but the fact that some currents have changed reality means that some of the bosonic fields have become purely imaginary. Actually, fixing $\beta = 1$, from table 2 we see that while all the fields are real for the AdS_6 case (following convention I), using convention II, $\mathcal{R}_{ab}$, $\mathcal{F}^r$ and $\widehat{\mathcal{F}} \equiv dA - mB$ are purely imaginary. Since the derivation of the supersymmetry transformation rules and the construction of the lagrangian are not affected by the presence of purely imaginary fields, we can take for this new theory (which has both real and imaginary fields) the same supersymmetry transformation rules and lagrangian of $F(4)$ AdS_6, but using Dirac conjugate spinors defined according to convention II. At this point we may rewrite the new theory in terms of real fields only. Of course, this has to be done preserving the R-symmetry group and the gauge invariance

$$B \longrightarrow B + d\lambda, \qquad A \longrightarrow A + m\lambda\,. \tag{B.1}$$

If $\mathcal{F}^r$ is pure imaginary we must define a real field strength $\widetilde{\mathcal{F}}^r = -i\mathcal{F}^r$; the only way to do it is to define

$$\begin{aligned} \widetilde{A^r} &= -iA^r \\ \widetilde{g} &= ig\,. \end{aligned} \tag{B.2}$$

[1] We recall that because of redefinitions (2.42), (2.50), there has been an interchange between convention I and II

Furthermore, in the limit $m = 0$, corresponding to the theory where $F(4)$ is contracted to a group containing as a bosonic subgroup ISO(1,5) $\otimes$ SO(4), to preserve the SO(4) R-symmetry group, the field A must be redefined as A^r, that is

$$\widetilde{A} = -iA\,. \tag{B.3}$$

Since the algebra closes for $g = 3m$ we are also forced to define

$$\widetilde{m} = im\,. \tag{B.4}$$

Finally, since we want to maintain the gauge invariance (B.1), taking into account (B.3), (B.4), we define

$$\widetilde{B} = -B\,. \tag{B.5}$$

Performing redefinitions (B.2)–(B.5) in the lagrangian we obtain that the kinetic term of the newvector field strengths $\widetilde{\widehat{\mathcal{F}}} \equiv d\widetilde{A} - \widetilde{m}\widetilde{B}$ and $\widetilde{F}^r$ have reversed sign with respect to $\widehat{\mathcal{F}}$ and $\mathcal{F}^r$, that is, they are ghosts, according to our previous finding. In addition we see that we do not have to perform any redefinition on the spin connection ω^{ab}, since according to the redefinition (B.4) on m (see equation (2.44)) ω^{ab} is defined as a real field.

We stress that, because of redefinition (B.4) the cosmological constant, in terms of $\widetilde{m}^2$ has changed sign with respect to the AdS_6 case we started from. This means that the new theory describes a dS_6 supersymmetric background.

To complete the discussion we have also to investigate whether the spin $\frac{1}{2}$ field χ_A and the dilaton field σ, that appear in the FDA out of the vacuum, do need appropriate redefinitions too. The superspace differential of the dilaton field σ is defined in the AdS_6 as follows (see references [6, 7]):

$$d\sigma = \partial_a \sigma V^a + \bar{\chi}_A \psi^A\,. \tag{B.6}$$

Following table (2) we see that using convention II the field σ should be pure imaginary; obviously this this is not the case. In fact, since the dilaton σ always appears as e^σ in the AdS_6 transformation laws, it would be impossible to impose the symplectic-Majorana condition (2.17) on the fermions if we replaced $e^\sigma \longrightarrow e^{i\sigma}$. This suggests that the dilatino χ_A should be redefined, in order to have a real scalar field σ, as follows:

$$\widetilde{\chi_A} = i\chi_A\,. \tag{B.7}$$

The same kind of reasoning applied to the matter fields, implies the following redefinitions:

$$\begin{aligned}\widetilde{A}^I &= -iA^I \\ \widetilde{\lambda}^I_A &= i\lambda^I_A\,.\end{aligned} \tag{B.8}$$

Equations (B.2)–(B.7) map the $F(4)$ theory with AdS_6 background into the $F(4)$ theory with dS_6 background. If we want to retrieve our previous results, obtained by explicit construction from the FDA (2.51)–(2.56), another inessential redefinition is needed: the Dirac conjugate of every spinor μ must be written as:

$$\widetilde{\overline{\mu}} = -i\overline{\mu}\,. \tag{B.9}$$

This is just because when we wrote the $F^t(4)$ algebra (2.16), we wanted it to have the same form for both $F^1(4)$ and $F^2(4)$. Therefore we had to choose, in order to have real fields, $\beta = -1$ for the $(t, s) = (2, 5)$ case, from which the AdS_6 theory comes from.[2]

The correctness of redefinition (B.7) can be checked considering that the variations under supersymmetry of the fermions $\delta\psi_A$, $\delta\chi_A$ and $\delta\lambda^I_A$ must satisfy the symplectic-Majorana condition (2.17) as the fermions do. One can in fact determine the reality of the coefficient in each term of the fermionic supersymmetry transformation rules (A.13), since it depends only on the number n of gamma matrices that it contains and on the choice of the convention I or II for defining the Dirac conjugate spinor.

In fact, let us evaluate the transposition and the hermitean conjugation of the product of n gamma matrices in $D = 6$ using equations (2.5), (2.12), (2.13):

$$\begin{aligned}
(\gamma_{a_1}\dots\gamma_{a_n})^T &= (-1)^n C^{-1}_{(-)}\gamma_{a_n}\dots\gamma_{a_1}C_{(-)}\\
(\gamma_{a_1}\dots\gamma_{a_n})^\dagger &= \alpha G^{-1}\gamma_{a_n}\dots\gamma_{a_1}G\\
\alpha_I &= 1 \quad \alpha_{II} = (-1)^n\,.
\end{aligned} \tag{B.10}$$

If we want that $\delta\psi_A$, $\delta\chi_A$ and $\delta\lambda^I_A$ satisfy the symplectic-Majorana condition (2.17) the following simple rule, descending from equations (B.10), must be obeyed: when n is even the current must appear in the transformation rule with a real coefficient in both cases I and II; when n is odd, there must be an imaginary coefficient if we use convention I and a real one if we use convention II.

This is exactly what we got for the AdS_6 case in reference [6, 7], and what we just obtained for the dS_6 by explicit construction and using redefinitions (B.2)–(B.8).

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